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FOUR INTRODUCTORY LECTURES IN Natural Philoſophy.

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LECTURE I.

OF THE
Rules of Philoſophiſing,
The eſſential Properties of Matter,
And Laws of Motion.

AS a knowledge of the operations of Nature, and their cauſes, is not only entertaining to the mind, but capable of being uſefully applied to practice in providing for the eaſe and convenience of life; it muſt ſeem ſurpriſing that Natural Philoſophy ſhould have made ſo very inconſiderable a progreſs in the world, as we find it had done about a Century ago. This muſt have been owing chiefly to the wrong methods by which it had been cultivated before that time. For, till then, philoſophers did not pay a proper attention to experiments, but employed their ſagacity in inventing ſyſtems and contriving Hypotheſes, by which they might explain the operations of Nature. They of latter times have more wiſely choſen to build their philoſophy on the ſolid foundation of facts and experience. The method, which they happily purſued, conſiſts in making a number of accurate experiments and obſervations, and from thence collecting what thoſe powers and principles of motion are which really obtain in Nature; and in explaining the phænomena, or natural appearances, from thoſe manifeſt principles aſſumed as cauſes; and in proving their explanations to be true by ſhewing, from mathematical reaſoning, that the cauſes aſſigned are adequate to the effects aſcribed to them, Theſe different methods of philoſophiſing, as one might reaſonably expect, were attended with very different ſucceſs. The former only amuſed the world with conjectures and philoſophical romances, the latter enlightened it by valuable diſcoveries.

The firſt who inſiſted on the neceſſity of uſing this method for promoting natural knowledge was Sir *Francis Bacon*, who flouriſhed about the beginning of the laſt century, and may juſtly be reckoned the founder of experimental philoſophy. This great man having taken an accurate and comprehenſive view of the ſtate of learning as it then ſtood, ſoon perceived the emptineſs of the philoſophy that had ſo long prevailed, which he ſays had been produced merely by the confidence of a few men, and received and admired only through the indolence and inattention of the reſt. And therefore, in his admirable *Treatiſe on the Advancement of Learning*, he propoſes, that men ſhould lay aſide all former Hypotheſes, and endeavour to raiſe a new ſtructure of philoſophy, not founded on uncertain opinions and ſpecious conjectures, but on truth and experience. The advantages, that might be expected by purſuing this method, he ſet forth in ſo ſtrong a light, that his plan was immediately approved of and adopted by the Learned in all parts of *Europe*; and ſince that time ſeveral men of great abilities, and particularly the famous Sir *Iſaac Newton*, have cultivated natural philoſophy according to his ſcheme, and enriched it with more important diſcoveries than all the foregoing ages can boaſt of. To give ſome account of theſe diſcoveries and to exhibit the experiments by which they are confirmed, is to be the buſineſs of the following lectures. But before I can proceed to this it is neceſſary to lay down certain rules, which will be of uſe in directing us how to deduce general concluſions from a number of experiments, and to inveſtigate cauſes from their effects. The rules delivered for this purpoſe by Sir *Iſaac Newton*, and which he himſelf ſo ſucceſsfully applied, are as follows:

RULE I.

More Cauſes of Natural Things are not to be admitted, than are both true and ſufficient for explaining their Phænomena.

THAT is, we are not to admit any thing as a cauſe whoſe exiſtence we cannot manifeſtly prove by experiments. And then we muſt not admit various cauſes of any effects when one is ſufficient. For all philoſophers agree that Nature does nothing in vain, and that is certainly done in vain by more cauſes which may be done by fewer. From hence follows a ſecond rule:

RULE II.

Of Natural Effects of the ſame Kind, the ſame Cauſes are to be aſſigned as far as can be done.

As, for inſtance, the ſame cauſe muſt be aſſigned, of reſpiration in a man and in other animals, of the deſcent of heavy bodies in different parts of the earth, of light in a culinary fire and in the ſun, of the reflexion of light in the earth and in the planets.

RULE III.

The Qualities of Bodies which cannot be increaſed or diminiſhed, and that agree to all Bodies on which Experiments can be made, are to be reckoned as Qualities of all Bodies whatſoever.

Thus if it be evident, from experiments and aſtronomical obſervations, that all bodies near the earth gravitate towards it in proportion to their quantity of matter, that the moon alſo gravitates towards the earth, and the waters of our ſea towards the moon, that the planets and the ſun gravitate towards each other mutually, that comets in like manner gravitate towards the ſun; and if the gravity of bodies, on which we can make experiments, can neither be increaſed nor diminiſhed; we muſt by this rule pronounce that gravity, or a mutual tendency towards each other, is a quality of all bodies whatſoever.

RULE IV.

In Experimental Philoſophy Propoſitions collected from the Phænomena by induction are to be deemed, notwithſtanding contrary Hypotheſes, either accurately true, or very nearly ſo; until other Phænomena occur, by which they may be rendered either more accurate or liable to Exceptions.

The meaning of this rule is, that againſt propoſitions founded on facts and experience, we are not to admit any exceptions that are taken merely from Hypotheſes or metaphyſical conſiderations; which we know have often miſled philoſophers in their inquiries, and ſeldom have been of any uſe to them.

These rules of philoſophiſing, and indeed all our attempts to enquire into the cauſes of natural things, depend entirely on this ſuppoſition; that the Univerſe hath been produced by the Will of an infinitely powerful and wiſe Agent, and that therefore all events, not depending on the will of man, are the effects of certain eſtabliſhed cauſes, made to act according to regular laws, the very beſt that could be deviſed, for preſerving the whole frame of Nature in the ſtate we now obſerve it to be. For in this caſe only can we hope to arrive at ſome knowledge of thoſe general cauſes, and their manner of acting, by carefully attending to the phænomena. Could we once ſuppoſe the Univerſe to have been the effect of chance, any endeavour to find out the cauſes of the common operations of Nature, or the laws by which they are regulated,

lated, would be a vain attempt and quite inconſiſtent with ſuch an Hypotheſis.

But experience fully evinces the truth of our firſt ſuppoſition. For philoſophers, directing their reſearches by the foregoing rules, have diſcovered, and plainly proved, that the phænomena, or natural appearances, are effected by the moſt ſimple and general cauſes, made to act with all the conſtancy, regularity and uſefulneſs, that could be expected in the works of perfect Power, Wiſdom and Goodneſs. They have alſo proved that theſe are but ſecondary cauſes, and ſuch as never could have produced the Univerſe out of Nothing, or could even have given its parts their firſt motions, or reduced them to the order they are now in.

Such diſcoveries therefore muſt amount to a ſtrict demonſtration, that all things were at firſt produced, and are ſtill governed, by the Will of an everliving powerful, wiſe and good Being; whoſe Influence cannot ſuffer any diminution through the greateſt diſtance of ſpace or length of time. So that the great truths of Natural Religion are the moſt evident concluſions, as well as the happieſt fruits, of Natural Knowledge.

HAVING given ſome account of the methods of philoſophiſing and the rules we are to obſerve, I proceed to conſider the Properties of Matter, and firſt ſuch as are eſſential to it.

AS the intimate Nature, or real Eſſence of Matter, is unknown to us, we can define it only by ſuch properties as we take to be eſſential to it, and without which we cannot conceive it to exiſt. I ſhall therefore ſay, that Matter or Body is a Subſtance extended, ſolid and perfectly inactive, or incapable of producing any change in its ſtate, whether of reſt or motion.

THE idea of Extenſion is ſo very ſimple and clear, that any attempt to explain it would probably render it more obſcure.

SOLIDITY is that property by which a Body is enabled to reſiſt and abſolutely prevent all others from coming into its place, till it has left it. By this property it is that Body is ſaid to fill Space, and upon this is founded the common Axiom; that it is impoſſible for two Bodies to be in the ſame place at the ſame time. Though ſome Bodies, as Fluids for inſtance, ſeem to give very little reſiſtance to others, yet this does not ariſe from any defect of ſolidity; but from their parts yielding to the leaſt impreſſion, and therefore being eaſily put out of their places. For if the moſt fluid Body be preſſed equally on all ſides, its reſiſtance will then be as ſenſible as that of the hardeſt. So likewiſe, though ſoft ſpongy Bodies may be compreſſed into a ſmaller ſpace than they uſually take up, yet they are nevertheleſs ſolid; for there is always a certain degree beyond which no force can compreſs them.

That Matter is capable of being put into Motion, our experience fully teſtifies. We have a power of moving our limbs according to the determination of our will, and we frequently ſee Bodies communicate Motion to each other by Impulſe. In what manner Motion is communicated, in the firſt caſe, we ſhall probably never be able to comprehend. In the ſecond caſe, we can eaſily conceive the power of communicating Motion to ariſe merely from Solidity. For when a Body in Motion ſtrikes another at reſt, which is not retained in its place by ſome force; ſince theſe Bodies, on account of their Solidity, cannot penetrate each others dimenſions, the ſtriking Body muſt protrude the other and move it out of its place. So that the communication of Motion, in this caſe, ariſes as neceſſarily from the Solidity of Bodies, as their having Figure, or being diviſible, does from their extenſion.

Every body muſt be terminated by by one or more ſurfaces, and therefore all Bodies muſt have ſome ſort of Figure; hence Figureability is reckoned an inſeparable property of Matter.

Extension neceſſarily implies Diviſibility, for whatever is extended muſt have Parts, and conſequently muſt be diviſible into theſe Parts. Diviſibility is therefore an inſeparable property of every particle of Matter, however ſmall. Hence we are neceſſarily led to affirm that Matter is diviſible *in infinitum*, the meaning of which is, that any particle of Matter is capable of being divided into parts, the number of which may be greater than any given number. As

Lines and Surfaces are extended, they alſo muſt be capable of an endleſs diviſion, as mathematicians have demonſtrated (*a*).

Although Matter is capable of being diminiſhed without end by a continued diviſion of its Parts, yet we muſt ſuppoſe that the ultimate and elementary Particles of which Bodies conſiſt, are ſuch as never were, nor probably can be, divided by any ordinary force. Sir *Iſaac Newton* judges (with great appearance of reaſon) that Matter in the beginning was formed in ſolid, maſſy, impenetrable, moveable Particles; which are incomparably harder than any porous Body compounded of them, even ſo hard as never to wear or break in pieces. For while the Particles (ſays this great Philoſopher) continue entire, they may compoſe Bodies of the ſame nature and texture in all ages. But ſhould they wear away or break in pieces, the nature of things depending on them would be changed. Water and Earth compoſed of old worn particles and fragments of particles would not be of the ſame nature

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(*a*) Tacquet's *Euclid*, *Lib.* iii. *Prop.* 16. *Cor.* 4.

nature and texture now with Water and Earth compoſed of entire Particles in the beginning. And therefore he concludes, in order that the nature of Bodies may be laſting, that the changes of corporeal things muſt conſiſt only in the various ſeparations and new aſſociations and motions of theſe permament Particles; compound Bodies being apt to break, not in the midſt of ſolid Particles, but where thoſe Particles are laid together and only touch in a few Points.

THE third Property, which I included in the definition of Matter, was its Inactivity, by which it not only perſeveres in its ſtate of Reſt or Motion, but muſt in ſome meaſure oppoſe every force that endeavours to make any change in its ſtate. For though a Body be in its own nature quite indifferent as to Reſt or Motion, yet when it is in either of thoſe ſtates it is not without ſome ſort of difficulty compelled to change it; and therefore muſt exert, as it were, ſome Force to preſerve itſelf in the ſtate it happens to be in. Hence this property of Matter is called its *vis inſita*, or, more expreſſively, its *vis inertiæ*. And this Force a Body never exerciſes but againſt ſome other which endeavours to cauſe a change in its ſtate. On which occaſion this Force may be conſidered either as a Reſiſtance or an Impulſe; as a reſiſtance, ſo far as, by endeavouring to perſevere in its ſtate, it yields with ſome difficulty to the Force impreſſed; and as an impulſe ſo far as, by yielding with ſome difficulty, it muſt make alſo ſome change in the ſtate of that Body which impreſſes the Force upon it. Reſiſtance is generally attributed to quieſcent Bodies, and Impulſe to thoſe that are in Motion. As we cannot conceive the *vis inertiæ* of the ſame ſolid Particle of Matter to be increaſed or diminiſhed, we muſt ſuppoſe that it is always the ſame in equal Quantities of Matter, and therefore that the *vis inertiæ* of any Body is proportional to its Quantity of Matter.

On theſe eſſential Properties of Matter is founded the Doctrine of Motion, which is juſtly called the Key of Nature, as without a true knowledge of that we can ſcarce account for any of her operations. Nothing therefore could ſo much have obſtructed Philoſophers in their purſuit of Natural Knowledge as their forming wrong notions concerning the nature of Motion.

Des Cartes held that all Space was equally full of Matter, and that there was always the ſame Quantity of Motion in the world, both which opinions have ſince been proved to be erroneous. Even the ſagacious *Kepler*, on whoſe Aſtronomical Obſervations and Diſcoveries the *Newtonian* Philoſophy is built, muſt have been inevitably hindered from ſucceeding in his various attempts to find out the phyſical cauſes of the Planetary Motions, by his having imagined that, from the Inertneſs of Matter, a Body put in Motion by any ſingle Impulſe would ſoon after ſtop of itſelf, and that therefore it required to have a Force continually impreſſed upon it to keep up its Motion, which is not at all neceſſary.

The Laws of Motion, or Laws of Nature as they are ſometimes called, which are immediately deduced from the eſſential Properties of Matter, are as follows.

LAW I.

Every Body muſt perſevere in its State of Reſt, or of uniform Rectilineal Motion, till by ſome Force it be compelled to change its State.

Every one will readily allow that a Body at Reſt muſt for ever continue ſo, unleſs ſome Force puts it in Motion. Now a Body, when put in Motion by a ſingle Impulſe, is not leſs paſſive in perſevering to move than it is in continuing to reſt. for if it be granted that a Body is unable of itſelf to make any change in its ſtate, it muſt be juſt as unable to deſtroy its own Motion, or to cauſe any change in its Velocity, or in the Direction of its Motion, as it is to change its ſhape, or to begin of itſelf to move from a ſtate of reſt. Some Philoſophers indeed ſeem to have thought that Bodies were naturally more diſpoſed to Reſt than to Motion; and probably they were induced to think ſo, by obſerving that all the Motions produced here on earth gradually decay, and at length ceaſe. But Bodies never ceaſe to move without our being as well able to aſſign the cauſe of their ſtopping, as the reaſon of their beginning to move. For when a Body is put in Motion, we find it does not ſtop, till its Motion is deſtroyed by the Reſiſtance of ſome other Body that it ſtrikes or rubs againſt.

LAW II.

Every Motion, or Change of Motion, in any Body muſt be proportional to, and in the Direction of, the Force impreſſed.

For ſince a Body, by reaſon of its Inactivity, cannot produce its own Motion or cauſe any change therein, it is evident that whatever Motion or change of Motion is generated in any Body muſt proceed entirely from the Force impreſſed, and conſequently muſt be proportional thereto, and muſt likewiſe be directed towards the ſame part with the Force impreſſed.

LAW III.

Reaction is always equal to Action, and contrary thereto; or, in other Words, the Actions of two Bodies on each other are always equal, and are exerted in oppoſite Directions.

Whatever preſſes or draws any thing is itſelf as much preſſed or drawn in an oppoſite direction. If I preſs a ſtone with my finger directly downwards, I find my finger as much preſſed by the ſtone, and that directly upwards. If a horſe draws a load by means of a rope, ſince that rope is equally ſtretched in oppoſite directions, by endeavouring to relax itſelf, it muſt urge the horſe towards the load as much as it urges the load towards the horſe, and muſt as much retard the progreſs of one as it promotes that of the other: for the force which the horſe exerts would carry him to a greater diſtance in the ſame time, were he freed from the incumbrance of the load, and therefore as much as his progreſs falls ſhort of that diſtance, ſo much is he in effect drawn back by the reaction of the load.

As the Laws of Motion cannot be properly confirmed by Experiments, till I have treated of thoſe Powers or Qualities of Bodies, which we conſider as the immediate cauſes of their ſeveral Motions. I ſhall employ the remaining part of this Lecture in explaining ſome terms which will frequently occur hereafter; and in laying down the method of computing the Proportions which the Velocities of Bodies, and alſo their Forces and Quantities of Motion bear to each other.

VELOCITY is that Affection of Motion by which a Body deſcribes a certain Space in a certain Time. For if I am aſked, what the Velocity of a Body is in any particular caſe? I muſt anſwer by telling the Space which that Body will deſcribe in a certain Time. And the Velocity in that caſe is always ſuppoſed to be uniform.

THE Velocity of a Body is ſaid to be equable or uniform, when it continues the ſame for the whole time during which the Body moves. And here it muſt be obſerved, that whenever I ſpeak of the Velocity of a moving Body, that Velocity is always ſuppoſed to be uniform, unleſs it is expreſsly ſaid to be otherwiſe.

IN computations relating to Forces, Motions, Times, Velocities, or Spaces deſcribed, we generally have occaſion to conſider the Proportions of theſe Quantities to each other, and not the abſolute Quantities themſelves. Therefore in ſuch computations, when any two of theſe quantities are compared together, we may ſubſtitute in their places any other two Quantities which have the ſame Proportion to each other, and which may be of a more convenient kind, ſuch as Numbers, Lines or Rectangles; and then thoſe Quantities are ſaid to be denoted or expreſſed by the others that are ſubſtituted for them. And therefore when one of thoſe Quantities is ſaid to be multiplied or divided by another, the meaning is that the Quantities, by which they are reſpectively expreſſed, are multiplied or divided one by the other: as when the Velocity of a Body is ſaid to be multiplied into the Time of its Motion, it is meant that the Numbers or Lines that expreſs theſe Quantities are multiplied by each other.

When one Quantity ſo depends on another that it always encreaſes or diminiſhes in the ſame Proportion in which that other is increaſed or diminiſhed, one of theſe Quantities is ſaid to be directly as the other, or directly proportional to the other. This manner of expreſſion is uſed for the ſake of brevity, for though but two Quantities are expreſſed, yet four at leaſt are always underſtood, and the expreſſion implies, that if each of the two Quantities be compared with another of its own kind, each of them will have the ſame Proportion to the one with which it is compared. For inſtance, when a Body moves for any given or determined Time, ſuppoſe a minute, it is evident that the Space it deſcribes in that Time will be greater or leſs in Proportion as its Velocity is greater or leſs, and therefore the Space deſcribed in that Time is ſaid to be directly as its Velocity, and this implies, that if two Spaces deſcribed in equal Times be compared, they will have the ſame Proportion to each other as the two Velocities have, wherewith they were deſcribed.

WHEN one Quantity depends on two others jointly, ſo that it increaſes or diminiſhes in the ſame Proportion in which either of them is increaſed or diminiſhed, while the other remains unvaried; then, if they are both varied, it will increaſe or diminiſh in the ſame Proportion in which the product of theſe Quantities is increaſed or diminiſhed: for it is evident that the product of theſe two Quantities depends on them jointly, in the ſame manner that the firſt Quantity was ſaid to do, becauſe it increaſes or diminiſhes in the ſame Proportion in which either of them is increaſed or diminiſhed while the other remains unvaried: and therefore, when any Quantity depends on two others jointly, ly, in the manner abovementioned, it will be directly proportional to their product. To illuſtrate this by an example; the Space which a moving Body deſcribes depends on its Velocity and the Time of its Motion jointly: for it is evident that the Space it deſcribes will be proportional to its Velocity, when the Time of its Motion remains the ſame, and to the Time of its Motion when its Velocity remains the ſame. Therefore in all caſes the Space which a Body deſcribes will be directly as the product which ariſes from multiplying its Velocity into the time of its Motion. Or, to expreſs this more fully, if the Spaces deſcribed by two Bodies be compared together, they will have the ſame proportion to each other which the two products have, that ariſe from multiplying the Velocity of each Body into the Time of its Motion. And theſe products will be denoted either by two Numbers or two Rectangles according as the Velocities and Times were denoted by Numbers or Lines. Now as the proportion of any two products in Numbers, or of two Rectangles, is compounded of the proportions of their reſpective Factors, or of their reſpective Sides; ſo the Spaces deſcribed by uniform Velocities are commonly ſaid to be to each other in a proportion compounded of the proportions of the Velocities and of the Times of the Motion.

When one Quantity ſo depends on another, that it increaſes in the ſame proportion in which that other is diminiſhed, and diminiſhes in the ſame proportion in which that other is increaſed; one of theſe Quantities is ſaid to be inverſely or reciprocally proportional to the other. For inſtance, when a Body deſcribes any given or determined Space, it is evident that the time, in which it deſcribes this ſpace, will increaſe or diminiſh in the ſame proportion in which its velocity is diminiſhed or increaſed. Therefore in this caſe, the time of the Body's Motion is ſaid to be inverſely as its velocity, and its velocity is ſaid to be inverſely as the Time of its Motion. Or to expreſs the ſame thing more fully, when two Bodies deſcribe equal Spaces, the Times of their Motions, when compared together, are to each other in the inverſe

verſe proportion of their Velocities; that is the Time of the ſlower Body's Motion is to the Time of the ſwifter Body's Motion, as the Velocity of the ſwifter Body to the Velocity of the ſlower.

WHEN one Quantity depends on two others jointly in ſuch manner that it is directly proportional to the firſt, when the ſecond is unvaried, and inverſely proportional to the ſecond when the firſt is unvaried; then, if they are both varied, this Quantity will be directly as the quotient, which ariſes from dividing the firſt of theſe two Quantities by the ſecond. For it is evident that the quotient of theſe two Quantities depends on them both, in the ſame manner that the firſt mentioned Quantity was ſaid to do. Becauſe the quotient of any two Quantities is directly proportional to the dividend when the diviſor is unvaried, and inverſely proportional to the diviſor when the dividend is unvaried. For inſtance, the Velocity with which a Body moves is directly as the Space it deſcribes, when the Time of its Motion is unvaried, and inverſely as the Time of its Motion, when the Space it deſcribes is unvaried; therefore when the Space which a Body deſcribes, and the Time of its Motion are both varied, its Velocity will be directly as the Quotient which ariſes from dividing the Space it deſcribes by the Time of its Motion. Or to expreſs this more fully, the Velocities of two moving Bodies, when compared together, will be to each other directly as the Spaces they deſcribe divided by the reſpective Times of their Motion. Now as the Proportion of any two Quotients is compounded of the direct Proportions of the Dividends and the inverſe Proportion of the Diviſors; ſo the Velocities of two Bodies are commonly ſaid to be to each other in a Proportion compounded of the direct Proportion of the Spaces deſcribed, and the inverſe Proportion of the Times of their Motions.

I HAVE choſen to illuſtrate theſe ſeveral methods of expreſſing the Proportions of Phyſical Quantities, by applying them to the Quantities of Velocities, Times and Spaces, becauſe our ideas of them are moſt clear and ſimple, and they are ſuch Quantities as occur moſt frequently in philoſophical and aſtronomical computations.

I SHALL now, from what has been ſaid, ſhew how we are to compute the Proportions which the Quantities of Motion or Forces of moving Bodies have to each other.

THE Quantity of Motion, or *momentum*, of a Body is that Force with which it moves. And this muſt always be proportional to the Force of that Impulſe which is able either to produce or deſtroy its Motion, for a Body cannot of itſelf produce its own Motion, or make any alteration therein, and therefore whatever Motion it has muſt be proportional to the Force that produces it. Now if a certain Impulſe or Force impreſſed on a Body, be able to give it a certain Velocity, then an Impulſe of a double or triple Force muſt produce a double or triple effect, that is, muſt give the Body a double or triple Velocity, and therefore in general the Velocity with which one and the ſame Body moves is proportional to the Force impreſſed; but the Force with which that

Body moves is alſo proportional to the Force impreſſed, and therefore muſt be proportional to its Velocity. But further, as the Force of a Body in Motion is compoſed of the Forces of all its Parts, which move with the ſame Velocity that the whole does, it is evident that the Force of a moving Body muſt be proportional to the number of its Parts, that is, to its Quantity of Matter, while its Velocity is the ſame. Since therefore the Forces of moving Bodies are directly as their Velocities when their Quantities of Matter are equal, and directly as their Quantities of Matter when their Velocities are equal, it follows, from what was ſaid before, that their Forces will, in all caſes, be to each other as the Products which ariſe from multiplying their Quantities of Matter into their reſpective Velocities.

Hence it follows that if the Forces of two Bodies be divided by their reſpective Velocities, the Quotients will expreſs their Quantities of Matter, or if their Forces be divided by their reſpective Quantities of Matter, the Quotients will expreſs their Velocities. This muſt appear evident, when we conſider that the Force or Quantity of Motion in each Body is expreſſed by the Product of its Velocity multiplied into its Quantity of Matter, and that when the Product of any two Quantities is divided by one of them, the other will be the Quotient.

It follows, from what has been proved, that two Bodies will move with the ſame Force if their Velocities be to each other inverſely as their Quantities of Matter. For in that caſe the Products will be equal, which ariſe from multiplying the Velocity of each Body into its Quantity of Matter. If therefore two Bodies be given, however unequal in their Quantities of Matter, they may be made to move with equal Forces; by making the Velocity of the leſſer Body exceed that of the greater, in the ſame Proportion in which the Quantity of Matter of the greater exceeds that of the leſſer,

LECTURE II.

OF THE
Several Kinds of Attraction,
And particularly of Coheſion.

AS the eſſential Inertneſs of Matter precludes all inanimate Bodies from having any Activity of their own or Propenſity to change their State, whether of Reſt or Motion; the various Motions of ſuch Bodies, in the ſeveral Operations of Nature, muſt ariſe from certain Forces impreſſed upon them by their Creator, either immediately, or by means of ſecondary Cauſes, which ultimately muſt be actuated by Him.

The Nature of theſe Forces muſt now be conſidered, and we ſhall find that the whole buſineſs of Philoſophy conſiſts, *Firſt*, in proving the Exiſtence of theſe Forces from ſuch ſimple and obvious Effects as cannot well be aſcribed to any other immediate Cauſe.

Secondly; in diſcovering the laws according to which theſe Forces are made to act, by carefully attending to the Nature of their Effects.

Thirdly; in aſſuming theſe Forces as Principles or Cauſes, and from thence, explaining the ſeveral Phœnomena; always proving our Explanations by mathematical Reaſoning, or other juſt methods of arguing, founded on the eſſential Properties of Matter and Laws of Motion.

Fourthly; in endeavouring to inveſtigate the Cauſes, from whence theſe Forces immediately ariſe, and in deducing Cauſes from Effects, till we come to the very firſt Cauſe, which certainly is not mechanical, that is, cannot conſiſt in any Properties of Matter or Motion. And though every true ſtep in this Philoſophy does not bring us immediately to the knowledge of the firſt Cauſe, yet it brings us nearer to it, and on that account is highly to be valued.

I SHALL now briefly enumerate the principal Forces by which Bodies are acted upon, and afterwards treat of each of them in their order, and the ſeveral Phœnomena that ariſe from them.

FIRST; The particles of Bodies, eſpecially of ſuch as are hard, muſt be made to cohere or ſtick together by ſome certain Force, ſince a conſiderable Force is neceſſary to ſeparate them from each other. For if the parts of a Body continued together merely from their *vis inertiæ*, and becauſe they are at reſt among themſelves, (as ſome have imagined) it could not then require a greater Force to ſeparate them from each other than it does to move the whole Body from its place. So that the *vis inertiæ* of Matter can never cauſe the parts of Bodies to ſtick together, and conſequently ſome Force muſt be impreſſed upon them to produce this effect, which Force is called the *Attraction of Coheſion.*

SECONDLY; If I let a Body go from my hand, tho' I do not give it any Impulſe, yet it will immediately move towards the center of the Earth; and if it be projected in any direction oblique to the Horizon it will not move on uniformly, and in a right line, as it ought to do by the firſt Law of Motion, but will be made to move in a curve till it reaches the Earth; which plainly ſhews that ſome Force continually acts upon it, to turn it from the rectilineal Courſe, which it would otherwiſe purſue. This Force is called *Gravity, or the Attraction of Gravitation.*

THIRDLY; if a load-ſtone and a piece of iron be ſuſpended near each other, or made to float on water, they will approach towards each other and ſtick together when they come into contact. Two pieces of iron, properly rubbed on a load-ſtone, will alſo come together in the ſame Manner, and in ſome Caſes they will recede or fly from each other and from the Magnet; which ſhews that a Force then takes place, that is of a contrary nature to the former One. Theſe two Forces are called *Magnetic Attraction and Repulſion.*

FOURTHLY; if amber, glaſs or wax be rubbed briſkly with a piece of dry leather, any light Bodies near them will be firſt drawn towards them, and then repelled from them. The Forces which act in this Caſe are called *Electric Attraction and Repulſion.*

THOUGH the beſt Philoſophers have agreed in calling theſe Forces by the names of Attraction and Repulſion, yet they intend thereby only to denote certain Cauſes or Powers, whatever they are, that produce well known Effects, and not to expreſs the Nature of theſe Cauſes or the Manner in which they operate. And as there is a manifeſt convenience in having names for theſe general Cauſes which muſt be ſo often mentioned, to object to the uſe of theſe words, as ſome have lately done, is but diſputing merely about names, and could ariſe only from not knowing the proper ſenſe in which theſe words are to be taken. We may alſo conſider theſe Forces, till their Cauſes are found out, as Powers or Qualities with which Bodies have been endued for carrying on the Operations of Nature.

The Attraction of Coheſion, tho' it has but very little effect, even at ſmall diſtances, yet is ſufficient to make the Parts of Bodies, when brought into cloſe contact, unite firmly together. Thus if two plates, made of any Matter that takes a fine poliſh, be preſſed together, they will cohere with a ſenſible Force, and much more ſtrongly if they be moiſtened by any liquid which excludes the air, and fills up their ſmall ſcratches and inequalities.

This kind of Attraction appears remarkably in thoſe Bodies that are tenacious or viſcid, ſuch as wax, pitch, or bird-lime, which, being ſoft, are eaſily brought into cloſe contact with other Bodies, and adhere ſtrongly to almoſt every thing they touch: and when a Body is very ſoft and viſcid, if it be ſtretched out, it will run back again and contract itſelf into a narrower compaſs, which plainly ſhews a tendency of the parts to each other.

The particles of Fluids tho' ſo eaſily ſeparated are not entirely void of this kind of Attraction, for the drops of all Fluids affect a round figure, which muſt ariſe from their particles running together ſo as to be all contained within the ſmalleſt dimenſions, as they are when the drop has a ſpherical ſhape; in like manner two drops of water or clean quickſilver, when brought into contact, run immediately into one drop. When a ſmall quantity of Quickſilver is poured on Paper, its drops ſtill retain their ſpherical ſhape, which ſhews that the Attraction of its Particles to each other is ſo great that their weight is not able to ſeparate them, and make them ſpread over the Paper as a drop of Water would do: but if the Quickſilver be laid on any clean metalline Body (except Iron) it will ſpread upon its Surface, its Particles being more attracted by the Metal than by each other.

The Attraction between Fluids and hard Bodies will further appear from the following obſervations.

Water always riſes ſomewhat about the ſides of the veſſel in which it is contained,

tained, and if the veſſel be very narrow the ſurface of the Water will be viſibly concave towards the middle. The ſame thing will happen when Mercury is contained in a veſſel made of any metal that attracts its particles more ſtrongly than they do each other; but in other veſſels the Mercury will be depreſſed round the edges and convex in the middle, its particles being more attracted by each other than by the ſides of the veſſel. Water riſes in the like manner round a Glaſs Bubble that floats on its ſurface, and when the veſſel is narrow, as the Water is concave towards the middle, it muſt riſe higheſt between the Bubble and the ſide of the veſſel neareſt to it, therefore the Attraction of the Water being ſtrongeſt on that part of the Bubble, will make it move towards the ſide of the veſſel, which it will do with an accelerated Motion. I know this has been aſcribed to an Attraction between the Bubble and the ſide of the veſſel, but that cannot be the caſe, for the Bubble, when it floats freely on the Water, will not be in the leaſt affected by the Attraction of a Glaſs Tube, or any other Body tho' held almoſt in contact with it; but if the Glaſs Tube be dipped in the Water, the Bubble will immediately run to it, which ſhews that it is affected only by the Attraction of the Water, which then riſes between it and the Tube. Beſides if the veſſel be filled till the Water riſes a little above the edge, the Bubble will quit the edge and move towards the middle, where the Water is then higheſt. And if two clean Glaſs-bubbles float near each other they will come together, becauſe the Water riſes higher between them than it does on their oppoſite ſides: but if the Bubbles be greaſed ſo that the Water may not riſe about them, they will not approach each other, for the attraction of the Bubbles themſelves is too weak even at ſmall diſtances to bring them together, and overcome the reſiſtance they meet with in moving through the Water: indeed when the floating Bodies are very large, their Attraction may be able to overcome this reſiſtance; and accordingly we find that large ſhips floating near each other in a calm are with difficulty kept from coming together.

All theſe Phœnomena, which are of the moſt ſimple kind, clearly point out a certain tendency in the minute particles Matter to each other, and conſequently prove the exiſtence of ſome Force which muſt be the cauſe of this tendency; and tho' we do not diſcover the nature of this Force, or its manner of acting, yet, having proved its exiſtence, we may aſſume it as a natural principle of Motion, and proceed from thence to explain other Phœnomena of a more complicated nature than thoſe before mentioned. Thus we may on this Principle account for the riſing of Fluids in Capillary Tubes, and for other effects of the like kind.

When the orifice of a ſmall Glaſs Tube, open at both ends, is dipped in Water, the little Ring or ſmall annular Surface of Glaſs that lies on the inſide of the Tube, and juſt over the Orifice, will ſtill draw up the Water that lies immediately under it, and make it aſcend into the Tube, and every Plate of Water, as it is drawn in, will raiſe up that which lies above it, until the weight of the Water raiſed is able to counterbalance the Force by which the Attracting Annulus endeavours to draw in more Water: if the Orifice of the Tube be $\frac{3}{25}$ of an inch in diameter, the Water will riſe to the height of about $\frac{1}{4}$ of an inch, and then remain ſuſpended. The thin Plate of Water that lies juſt over the loweſt Annulus, and every other Plate of the elevated Water, muſt be drawn both upwards and downwards with equal Forces, becauſe it has equal attracting Surfaces above and below it, and therefore the whole Column of elevated Water which lies above the loweſt Annulus being drawn equally in oppoſite directions, may be conſidered as not at all affected by the Attraction of the Glaſs, and muſt preſs with its whole weight on the Water which is retained at the Orifice of the Tube, by the Force of that loweſt Annulus that has no other Surface below it to counteract its Attraction.

From this way of reaſoning it ſeems evident that the aſcent and ſuſpenſion of Water in theſe Tubes are entirely owing to the Action of the very loweſt Glaſs Annulus that lies juſt over the Orifice, and not to that of the Annulus which lies over the elevated Water, or of any other part of the internal Surface of the Tube, as hitherto has been imagined. However as what I have advanced is contrary to the received opinion, it may be neceſſary to confirm it by the following Experiment.

WHEN the Water has aſcended into a ſmall Glaſs Tube, take it up, wipe off the Drop that hangs at the end of it, and invert the Tube; the ſmall Column of included Water, being then drawn equally by the Glaſs in oppoſite directions, will deſcend by its own weight, and not ſtop till it arrives at the other Orifice; which ſhews that the Force which keeps it ſuſpended lies there. And if, while the included column is deſcending, the loweſt end of the Tube be dipped in Water and taken up, a ſmall Plate of Water will be retained at its Orifice, and the included Column will remain ſuſpended; now this Column of Water muſt, by means of the intermediate Air, preſs with its whole weight on the ſmall Plate of Water at the Orifice, and this Plate can be retained there by no other Force than the Attraction of the Glaſs Annulus that lies juſt within the Orifice of the Tube: and therefore it appears that this Attraction alone is ſufficient to ſuſtain, and conſequently to raiſe all the Water that ſeems ſpontaneouſly to aſcend into a Glaſs Tube.

THIS attracting annular ſurface on the inſide of the tube is of a very ſmall breadth, becauſe the action of the glaſs extends to but a very ſmall diſtance, and as it is of the ſame breadth in different tubes, its quantity, or the number of attracting particles, muſt be proportional to the diameter of the tube, conſequently in cylindrical tubes the quantities of Water raiſed muſt be as their diameters, and therefore the altitudes of the cylindrical columns of Water raiſed will be inverſely as their diameters; becauſe when cylinders are to each other as their diameters, they muſt have their heights inverſely proportional to their diameters; and this is the reaſon why Water riſes higher in narrow tubes than in wide ones. But having meaſured, as accurately as I could, the diameters of ſeveral tubes and the heights to which the ſame fluid roſe in each, I found in a great many trials that theſe heights were not exactly in the inverſe proportion of the diameters, for in the narrow tubes the Fluid always roſe higher than it ſhould do according to that proportion.

And indeed there ſeems to be a plain reaſon for this, for in narrow tubes the oppoſite attracting particles being cloſer together act more in conjunction in drawing up the Fluid, and being alſo nearer to the middle Parts of the Fluid, they muſt on that account act more forcibly in raiſing and ſuſtaining it.

When a drop of water or any thin oil, is included in a ſmall conical glaſs tube, open at both ends, it will run with an accelerated Motion to the narroweſt end, although that end be raiſed ſomewhat higher than the other; the reaſon of which may be thus ſhewn. Let the * lines A B and C D repreſent the ſides of a ſmall conical tube and let P be a particle in the included drop, from which draw P E and P F perpendicular to the oppoſite ſides of the tube; theſe ſides muſt exert their attractions on the particle P in the directions of the lines P E and P F, and therefore theſe Forces (as ſhall be ſhewn when I come to treat of the compoſition of Motion) will by their joint action make the particle P move in the direction of a line P G, drawn thro' the angle E P F and pointing towards the ſmalleſt end of the tube, and conſequently the whole drop muſt move towards that end.

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* See *Fig.* 1.

For the ſame reaſon that Water riſes in ſmall cylindrical tubes, it will riſe between two glaſs planes ſet parallel to each other, and kept at a ſmall diſtance aſunder. And ſince the quantity of attracting glaſs ſurfaces will in this caſe be ſtill the ſame, whatever be the diſtance between the planes, the quantity of elevated Water muſt ſtill be the ſame. Therefore in whatever proportion the diſtance between the planes is diminiſhed, the height of the elevated Water muſt be increaſed. And therefore, if the edges of the Planes are made to meet on one ſide, and the oppoſite edges are kept at a ſmall diſtance from each other, the elevated Water will have different heigths at different diſtances from the concourſe of the Planes; ſo that the ſurface of the elevated Water will form a regular curve, riſing from the ſurface of the Water in the veſſel towards the concourſe

courſe of the Planes. Let the * line A B repreſent the concourſe of two glaſs Planes, whoſe oppoſite edges at C O are kept at a ſmall diſtance from each other, and let A C be the ſurface of the Water in which the lower edges of of the Planes are immerſed; the curve line K L M N will repreſent the ſurface of the Water elevated between the Planes by their attraction. Now if any two columns of this elevated Water, as D L and F M be taken of very ſmall but equal breadths as D L and F G, theſe columns will contain equal quantities of Water, being ſuſtained by equal portions of the attracting Planes; therefore the heights of theſe columns muſt be inverſely as their thickneſs, that is D L muſt be to F M as the interval between the Planes at F is to their interval at D, but theſe intervals are to each other as A F and A D their diſtances from the concourſe of the Planes, therefore the heights of any two columns of the elevated Water are to each other inverſely as their diſtances from the concourſe of the Planes.

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* See *Fig.* 2.

Let two well poliſhed Planes of Glaſs be laid together, ſo that the loweſt may be parallel to the Horizon, and the other may make with it a ſmall Angle of about a quarter of a Degree; the inward ſides of the Planes being well moiſtened with any thin Oil, let a Drop of the ſame Oil be put in with a Feather, ſo that it may touch both Planes, it will begin to move towards their Concourſe, for the ſame reaſon that it would move towards the narroweſt end of a ſmall conical Tube; and notwithſtanding the ſide where the Planes meet be ſomewhat elevated, the Drop will continue to aſcend, until the elevation of the Planes becomes ſo great that the weight of the Drop, which is always proportional to that elevation, will be ſufficient to counteract the Force by which the Drop is drawn towards the concourſe of the Planes.

Now by obſerving the elevations of the Planes that are neceſſary to ſtop the motion of the Drop when at different diſtances from the concourſe of the Planes, it has been found that equal quantities of the Glaſs Surfaces, taken at ſeveral diſtances from the concourſe of the Planes, exert attractive Forces on the included Drop that are to each other in the inverſe proportion of thoſe diſtances, that is, the Force with which the Drop is attracted by the Planes, when placed at the diſtance of ſix inches from their concourſe, is but one half of the Force with which it would be attracted at three inches, and but a ſixth part of the Force with which it would be attracted at the diſtance of one inch from the concourſe; ſuppoſing the Drop not to ſpread but to be ſtill touched by equal ſurfaces of the Glaſs: (*b*) and therefore the Attraction of the Glaſs

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(*b*) It is evident that the Quantity of Surface over which the Drop ſpreads muſt increaſe as the thickneſs of the Drop diminiſhes, that is, as its diſtance from the concourſe of the Planes diminiſhes; therefore the ſame Drop at three inches from the concourſe will ſpread over double the Quantity of Surface that it does at ſix inches from the concourſe. Now experiments ſhew that the elevations of the Planes neceſſary to ſtop the Motion of the Drop at theſe diſtances are to each other as four and one, and therefore the Attraction of the double Surface at three inches from the concourſe is four times greater than the Attraction of the ſingle Surface at

Glaſs Surfaces grows greater in the ſame proportion as their diſtance from each other grows leſs; conſequently where the diſtance between them is exceeding ſmall their Attraction muſt be exceeding great.

SIR *Iſaac Newton*, who mentions this Experiment, in the Queries at the end of his Opticks, computes that where the thickneſs of the Oil between the Planes is about the two millioneth part of an inch, the Attraction will be ſo great as to be able to ſuſtain, within a circle of an inch in diameter, the weight of a Column of Water two or three furlongs in length, and where it is of a leſs thickneſs the Attraction will be proportionally greater, and will continue to increaſe until the thickneſs does not exceed that of a ſingle particle of oil. There are therefore agents in nature able to make the particles of Bodies ſtick together with very ſtrong attractions.

IN all the experiments already mentioned, the Phœnomena happen exactly in

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at ſix inches from the concourſe; conſequently the Attractions of equal Surfaces at thoſe diſtances are to each other as two and one.

in the ſame manner *in vacuo*, that they do in the open air, and therefore they cannot be owing to the preſſure of the atmoſphere; but muſt neceſſarily be aſcribed to ſome power which cauſes the particles of the Fluid to tend more to the Glaſs than to each other. And if any Fluid be employed in theſe Experiments whoſe particles tend more to each other than to the Glaſs, the effects will then be the reverſe of the former ones. Now Mercury is ſuch a Fluid, for tho' its particles are attracted by Glaſs (as appears from their ſticking to the back of a looking-glaſs) yet they are more ſtrongly attracted by each other; and therefore when a ſmall Glaſs Tube is immerſed in Mercury it does not riſe ſo high within the Tube as it is in the veſſel: and when two Glaſs Planes, that make a ſmall angle with each other, and ſtand perpendicular to the horizon, are immerſed to ſome depth in a veſſel of Mercury, it will be higheſt between the Planes where they are fartheſt aſſunder, and loweſt towards their concourſe; which is the reverſe of what happens when Water is included between thoſe Planes.

If clean Mercury, well purged of air, be carefully poured into a clean Glaſs Tube, of about 70 inches long, ſo that its parts may be every where contiguous to each other and to the Glaſs, it has been found upon inverting the Tube, that the whole Mercury would remain ſuſpended. Now the preſſure of the Atmoſphere could only ſupport about 29 or 30 inches of Mercury, and therefore ſome other agent muſt have ſuſtained the reſt, not by preſſing the Mercury into the Glaſs, but by making its parts adhere to the Glaſs and to one another; for upon any diſcontinuation of parts, made either by bubbles intervening or by ſhaking the Glaſs, the whole Mercury falls down to the height of 29 or 30 inches,

It ſeems to be by the ſame attractive Force, with which ſmall pipes and capillary tubes are endued, that ſponges and other dry porous Bodies draw in Water, and the glands in the Bodies of Animals, according to their ſeveral natures and diſpoſitions, take in various juices from the Blood; and Plants receive nouriſhment from the Earth, the ſmall Tubes whereof their roots are compoſed, taking in different juices according to their different conſtitutions; juſt as the ſame capillary Tube of Glaſs acts differently on different liquors, which therefore will not all aſcend in it to the ſame height; for Water, though heavier than Spirit of Wine, will aſcend near twice as high in the ſame Tube, which ſhews it is twice as ſtrongly attracted by the Glaſs. It has been alſo obſerved, that Tubes made of different kinds of Glaſs will attract the ſame Liquor with different Forces, and cauſe it to aſcend to different heights.

MANY other Operations of Nature ſeem to be carried on merely by means of this Attraction that obtains among the minute particles of Bodies; particularly moſt of the Operations in Chymiſtry ſeem ſcarce explicable on any other principle. But as that ſubject is not within my province I ſhall enter into it no further than to give ſome account of that common Operation, the Solution of

Bodies in Fluids, which I ſhall have occaſion to make uſe of hereafter in explaining certain Phœnomena.

When the Particles of a Body, ſurrounded by a Fluid, are leſs ſtrongly attracted by each other than by the Fluid, they muſt ſeparate from each other and join themſelves to the Particles of the Fluid, and remain ſuſpended therein. Thus various Salts are diſſolved in Water, and Metals and other Bodies in the liquors that are reſpectively adapted to their Solution.

Upon the various degrees of Attraction between different Subſtances depend almoſt all the Phœnomena which Chymiſtry exhibits, and therefore it has been found of great uſe to have Tables compoſed wherein thoſe mutual Attractions are ſet down according to the different degrees in which they are found to obtain between ſeveral Bodies. Theſe are called tables of *Affinities*, or *Elective Attractions*, and as this is a curious ſubject, I ſhall illuſtrate it by the following inſtance.

When Silver is diſſolved in Aqua-fortis, which is the acid Spirit of Nitre, if Filings of Copper be thrown into the Solution, the Aqua-fortis will diſſolve the Copper and quit the Silver, which will be precipitated to the Bottom in the Form of a white Powder: if then Filings of Iron be thrown in, they will in like manner be diſſolved, and the Copper will fall to the bottom. And laſtly, if Salt of Tartar, or any other alkaline Salt be thrown in, the Aqua-fortis will unite with it and diſmiſs the Iron. This acid Spirit has therefore a greater affinity, or a ſtronger Attraction to the alkaline Salt than to any of the Metals, and it attracts the Iron, the Copper and the Silver with different degrees of ſtrength; and very ſtrong muſt theſe Attractions be, ſince they are able to ſeparate from each other the Particles of ſuch hard Metals.

From hence we may ſee the nature of that Operation of turning Iron into Copper (as it is called) by laying bars of Iron in Water that ſprings from a Copper-mine, and which is impregnated with Particles of Copper diſſolved in a mineral acid. This is by no means a tranſmutation of one metal into another, but the acid having a greater affinity with the Iron than it has with Copper, corrodes and diſſolves the Iron, and leaves the Particles of Copper in its place, which adhere together in ſmall ſcales; ſo that after ſome time the Bar is found incruſted with Copper, and moſt part of the Iron is carried off by the Water.

These different degrees of Attraction between the minute Particles of Matter ſeem to be the immediate cauſe of all the ſeveral degrees of hardneſs and ſoftneſs that we find in different Bodies; for the ſtronger the Attraction, the more firmly muſt the Particles cohere, and the harder muſt the Body be which they compoſe. But how far hardneſs and ſoftneſs may depend alſo on the ſize and ſhape of the ultimate and conſtituent Particles of Bodies we ſhall probably never be able to determine, becauſe they are too minute to fall under the cognizance of our ſenſes.

In many caſes when the Particles are placed without the ſphere of each other's Attraction, a repelling Force prevails among them; for two poliſhed Plates of Marble, which by immediate contact cohere together, are with difficulty brought into contact, and the object-glaſſes of long Teleſcopes lie upon one another without touching, as appears by the colours which the air between them exhibits. From the ſame repelling power it ſeems to be that Flies walk upon the Water without wetting their feet, and that the Particles of dry Powders are with difficulty made to touch one another ſo as to ſtick together. The Particles of Fluids, when ſeparated by heat, are turned into an elaſtic ſteam, which, if confined and ſufficiently heated, will burſt the ſtrongeſt veſſel. This repulſive Force ſeems alſo to be the cauſe of the emiſſion and reflection of the rays of light and of ſeveral other Phœnomena that ſhall be taken notice of in their proper places.

LECTURE III.

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OF GRAVITY,

OR THE

Attraction of GRAVITATION.

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THE ſecond kind of Attraction I propoſed to treat of was Gravity, or that Force which cauſes all Bodies near the earth, to tend towards its centre; and which prevents the earth's rapid Motion round the axis from the throwing off the Bodies that lie on its ſurface, as it certainly would do were they not retained there by ſome very potent principle.

As the drops of all Fluids affect a round Figure by the mutual tendency of their Parts to each other; ſo the ſurface of our ſea muſt be ſpherical, ſince the waters every where tend to the centre; and the prominences of the land above the ocean, are ſo very ſmall with reſpect to the earth's magnitude, that the whole may be conſidered as a ſphere. And ſuch it muſt appear to a ſpectator placed at a proper diſtance, juſt as the moon appears ſpherical to us, tho' a good teleſcope diſcovers greater inequalities on her Surface, than are to be met with on the Surface of the earth.

The Attractions of Coheſion and Gravity ſeem to be general Forces affecting all Bodies, for their particles cohere together when brought into cloſe contact, and the Bodies themſelves are all equally affected by the Force of Gravity. There is however this difference between them, the coheſion of Bodies may be diminiſhed or deſtroyed, and when their particles are ſeparated there is ſome natural

Force that makes them repel each other; whereas there is not any Body whoſe abſolute Gravity can be in the leaſt diminiſhed. This Force by which a Body tends to the centre of the earth is commonly called its Weight, and the Weights of Bodies at equal diſtances from the centre are always proportional to the quantities of ſolid Matter which they contain; for all Bodies whatever, being let fall from the ſame height, deſcend with equal velocities, provided they meet with no reſiſtance from the air; as will appear from the following experiment. If a piece of gold and a feather be let fall from the top of a tall exhauſted Receiver at the ſame inſtant, they will both come to the bottom at the ſame time.

Since therefore the attractive Force of the earth cauſes all Bodies, that fall freely, to deſcend with the ſame velocity; it follows that this Force muſt act equally on the particles of all Bodies whatever; and therefore it muſt act on every Body in proportion to its number of Particles, that is, in proportion to its quantity of Matter or its *vis inertiæ*;

for otherwiſe it could not cauſe all Bodies to move downwards with the ſame velocity. Conſequently the weights of different Bodies, at equal diſtances from the centre of the earth, or the Forces with which they deſcend freely, or endeavour to deſcend, are to each other as their quantities of Matter. The ſame thing may be proved thus; when Bodies begin together to deſcend freely from a ſtate of reſt, their quantities of Motion acquired in the ſame time muſt be proportional to the moving Forces, that is to their weights, but ſince they deſcend with equal velocities, their quantities of Motion are alſo proportional to their quantities of Matter; therefore their weights are likewiſe proportional to their quantities of Matter. Hence it is that the quantity of Matter in any Body may be juſtly eſtimated by its weight or the Force with which it endeavours to deſcend.

The attractive Force of the earth is found not to act equally on Bodies at all diſtances from its centre. Sir *Iſaac Newton* has demonſtrated, that this Force above the ſurface of the earth decreaſes in the ſame proportion that the ſquare of the diſtance from the centre increaſes, that is, the Forces with which the earth acts on Bodies to put them in Motion, at different diſtances above the ſurface, are reciprocally as the ſquares of their diſtances from the centre. So that if a Body at the ſurface of the earth, (whoſe diſtance from the centre is about 4,000 miles) weighs four pounds and by the Force of Gravity falls thro' 16 feet in a ſecond of time; it will at double that diſtance weigh but one pound, and will fall thro' but four feet in a ſecond of time. He has likewiſe proved that the Gravity of a Body at any place below the ſurface of the earth, is directly proportional to its diſtance from the centre; that is, a Body which at the ſurface weighs one pound will, within the earth at the diſtance of two thouſand miles from the centre, weigh but half a pound, and at the diſtance of one thouſand miles but a quarter of a pound, and ſo on till at the centre it loſes all its weight. The attractive force of the earth when conſidered as acting differently on Bodies at different diſtances from the centre is called its accelerating Force, and is meaſured by the velocity it is able to generate in the Bodies in a given time.

Tho' ſtrictly ſpeaking the accelerating Force of Gravity decreaſes in the manner above-mentioned, yet where the diſtances from the Surface are inconſiderable with reſpect to the Earth's Radius, (as are all the diſtances to which we can project Bodies) the Force of Gravity may be looked upon as equal at all thoſe diſtances. And on this ſuppoſition are founded the reaſonings of *Gallileo*, *Torricellius*, *Huygens* and other Naturaliſts, concerning the deſcent of heavy Bodies, and the ſeveral Theorems are formed which relate to the acceleration of falling Bodies, the ſpaces deſcribed by them, the times of the fall and the velocities thereby acquired.

If the Force of Gravity remains unvaried, the Motion of a Body, falling freely from a ſtate of reſt, will be uniformly accelerated; that is, the increments of the velocity will be equal in equal times. For let us ſuppoſe the time of the deſcent to be divided into a number of equal parts indefinitely ſmall; in each of theſe the Force of Gravity muſt make equal impreſſions on the Body to carry it downwards, and conſequently muſt by each impulſe generate an equal velocity in the falling Body; and ſince all the velocities are in the ſame direction, the laſt acquired muſt be ſtill added to the former, and therefore the velocity of the falling Body will be uniformly accelerated.

It follows alſo that, at the end of any time, the whole velocity generated muſt be as the number of Impulſes received from Gravity, that is, as the number of the indefinitely ſmall parts of time taken from the beginning of the fall, and therefore the velocity acquired at the end of any time by a Body, falling freely from a ſtate of reſt, will be as the time of its deſcent from the beginning of the fall.

The ſpace which a Body falling freely from a ſtate of reſt deſcribes in any time taken from the beginning of the fall, may be expreſſed by a right angled triangle * A B C, whoſe Baſe A B denotes the time of the fall, and the perpendicular B C the velocity acquired at the end of that time.

For let there be drawn in the triangle the lines H I, F G and D E parallel to B C, then ſince theſe lines H I, F G, D E and B C are to each other as A H, A F, A D and A B, and ſince B C denotes the velocity acquired in the time A B, the lines H I, F G and D E muſt denote the velocities reſpectively acquired in the times A H, A F and A D, becauſe the velocities are as the times of the Motion. Now if we ſuppoſe the whole time of the Motion to be divided into a number of very ſmall and equal parts, as A H, H F and F D, we may then conſider the velocity of the Body as uniform during each of theſe parts of time; and ſince the ſpace deſcribed by any uniform velocity is expreſſed by a rectangle contained under lines proportional to the time of the Motion and the velocity, the ſpaces deſcribed in the ſeveral ſmall parts of time A H, H F and F D will be expreſſed by the little rectangles A H I K, H F G L

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* See *Fig.* 3.

HFGL and FDEP, and thus the Space deſcribed in the whole time A B will be expreſſed by the ſum of all the little rectangles that are formed in this manner by multiplying each ſmall part of the time into the velocity with which the Body then moves. But when we ſuppoſe thoſe equal parts, into which the whole time A B is divided, to be indefinitely ſmall, and their number indefinitely great (as we ought to do if we conſider the velocity as uniform in each of thoſe parts of time) then the ſum of all the rectangles will differ from the triangle A B C by a quantity leſs than any given one, and conſequently the Space deſcribed in the whole time of the Motion will be expreſſed by the triangle A B C.

From hence we may eaſily deduce, as corollaries, all the other properties of an uniformly accelerated Motion. Firſt, the Space which a Body falling freely deſcribes in any time, taken from the beginning of the fall, is one half of the Space it would deſcribe in the ſame time if it moved uniformly with the velocity acquired at the end of the fall.

For when, in the right angled triangle A B C, the ſide A B denotes the time of the fall, and B C, the laſt acquired velocity, the Space deſcribed in the time of the fall will be expreſſed by the triangle A B C; but the ſpace deſcribed in the ſame time by a Body moving uniformly with the velocity B C will be expreſſed by the rectangle A B C O of which the triangle is one half.

SECONDLY; The Spaces, deſcribed by a Body falling freely from a ſtate of reſt, are to each other as the Squares of the times from the beginning of the fall, or as the Squares of the velocities acquired in thoſe times. For it appears from what has been ſaid, that the Spaces deſcribed by a falling Body in the times denoted by A B and A D will be in the ſame proportion with the triangles, A B C and A D E; but theſe triangles being ſimilar, their areas are to each other as the Squares of their homologous ſides; that is, as the Squares of the lines A B and A D, which expreſs the times of the fall, or as the Squares of the lines B C and D E, which expreſs the velocities acquired in thoſe times: Wherefore the Spaces deſcribed in different times by a falling Body are to each other as the Squares of the times from the beginning of the fall; or as the Squares of the velocities acquired at the ends of thoſe times.

To confirm this by an Experiment, let a weight of eleven hundred grains fall from the height of three inches, ſo as to ſtrike one end of a balance, its Force will be juſt ſufficient to raiſe a pound weight, hung at the other end of the balance, to a ſmall height; but in order to raiſe weights of two or three pounds the ſame body muſt fall reſpectively from the heights of twelve and twenty-ſeven inches, for leſs heights will not be ſufficient.

The forces wherewith the deſcending Body ſtrikes the end of the balance are proportional to the weights they are able to raiſe; but the Forces with which the ſame Body ſtrikes are alſo proportional to its velocities; wherefore the velocities acquired by the falling Body are, in this caſe, as the weights raiſed, that is, as one, two and three; and therefore the Squares of the velocities are as the ſquares of theſe numbers,

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that is, as one, four and nine; but the Spaces three, twelve and twenty-ſeven are alſo in the ſame proportion as one, four and nine, and therefore the Spaces deſcribed are as the Squares of the velocities acquired by a Body in falling through them.

Hence it follows that the Spaces deſcribed by a falling Body in the ſeveral moments of time taken ſeparately and in their order from the beginning of the fall, are to each other as the odd numbers, one, three, five, ſeven, and ſo on, taken in their natural order; for, where the times of the fall are one, two, three and four moments, the Spaces deſcribed will be as the Squares of thoſe times reſpectively, that is, as one, four, nine and ſixteen; therefore the Spaces deſcribed in the ſecond, third and fourth moments, taken ſeparately, will be as the differences between the numbers one, four, nine and ſixteen reſpectively, which are three, five, ſeven, nine, and ſo on.

As the Motion of a Body falling from a ſtate of reſt is uniformly accelerated, ſo likewiſe the Motion of a Body thrown directly upwards is uniformly retarded. For the ſame Force of Gravity which conſpires with the Motion of a falling Body, acts in direct oppoſition to the motion of ſuch as aſcend, and therefore, in whatever manner it accelerates one it muſt retard the other. Whence it follows that a Body thrown directly upwards will continue to aſcend for a time equal to that, in which, by falling from a ſtate of reſt, it would acquire the ſame velocity with which it is thrown up. For ſince the action of gravity is conſtant and uniform, in whatever time it generates any velocity in a falling Body, in the very ſame time it muſt deſtroy that velocity in a riſing Body, and therefore the time of the aſcent muſt be equal to that of the fall. Hence if a Body be thrown directly upwards with a velocity equal to that which it has acquired by falling from any height, it will aſcend to the ſame height before it loſes all its velocity. For ſince of theſe equal velocities one is generated and the other deſtroyed in the ſame manner by the uniform action of the ſame Force, and in the ſame time, it is manifeſt that the ſum of the velocities wherewith the body moves, during its deſcent and its aſcent, muſt be equal, and conſequently the ſpaces deſcribed in its deſcent and in its aſcent muſt alſo be equal.

In order to determine the abſolute Force of Gravity at the ſurface of the Earth it is neceſſary to know thro' what Space a Body will fall in a given time. And by very accurate experiments made with Pendulums, which will be mentioned hereafter, it has been found, that a Body, near the ſurface of the Earth, falling from a ſtate of reſt, will deſcend through the Space of about ſixteen feet and one inch in a ſecond of time, that is, in the ſixtieth part of a minute: from whence it appears that the Force of Gravity by acting on any Body only for one ſecond of time will generate in it ſuch a velocity as, being continued uniform, would carry it thro' two and thirty feet and two inches in one ſecond, or thro' very near twenty two *Engliſh* miles in an hour.

I must obſerve here that Bodies do not deſcend in all climates at the rate of ſixteen feet and an inch in a ſecond of time, but in ſuch places only as are in, or near, the Latitude of forty-nine degrees. In places more diſtant from the Æquator, the deſcent is quicker, and ſlower in thoſe that are nearer to it. For it has appeared, from a great number of obſervations, that the Force of Gravity is leaſt under the Æquator, and that it increaſes continually as we approach towards the Poles, where it is greateſt of all.

THE chief cauſe of this difference is, the rotation of the Earth round its Axis, by which all Bodies on the Surface acquire a centrifugal Force; and, as I obſerved before, would actually fly off, if they were not retained by their Gravity. (*c*) Now as all theſe Bodies revolve uniformly round the Earth's Axis in four and twenty hours, either in the Æquator or in circles parallel to it, their velocities, and conſequently

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(*c*) By the centrifugal Force, is meant that force which we ſee makes Bodies, lying looſely on a wheel or a globe, fly off when it revolves ſwiftly round its Axle; and this ariſes from their tendency to move on in a right line according to the firſt law of motion.

quently their centrifugal Forces, muſt be as the Peripheries of the Circles they deſcribe; and therefore as the Æquator is the greateſt of all thoſe Circles, and the others grow leſs and leſs as they are more and more diſtant from the Æquator, the centrifugal force of Bodies muſt be greateſt in the Æquator, and be continually diminiſhed towards the Poles, till at laſt it vaniſhes at the Polar points. As this force acts in oppoſition to Gravity, it muſt of courſe diminiſh it, and this diminution muſt be greateſt in the Æquator, and grow leſs and leſs in the approach towards the Poles.

There is alſo another reaſon why the Force of Gravity ſhould be more diminiſhed by the centrifugal force at the Æquator than in any other place, which is this. The centrifugal Force always cauſes a Body to tend from the centre of the Circle in which it moves; and therefore it muſt act in direct oppoſition to the Force of Gravity at the Æquator, whoſe center is the ſame with that of the Earth; but as the centers of all the Circles parallel to the Æquator are at ſeveral diſtances from the center of the Earth, in every ſuch Circle the centrifugal Force acts obliquely againſt the Force of Gravity, and the more ſo as the Circle is nearer to the Pole, and therefore cannot diminiſh it ſo much as it does at the Æquator.

SIR *Iſaac Newton* has proved, by a moſt ingenious method of computing, that Bodies at the Æquator loſe $\frac{1}{230}$ part of the weight which they would have if they were placed at the Poles; and has ſhewn that the Æquatorial diameter of the Earth exceeds the Polar diameter, or the Axis round which the Earth turns, by about ſeventeen *Engliſh* miles and $\frac{1}{10}$; for if the Æquatorial parts of the Earth where the Waters are lighteſt, were not ſo much higher than the Polar parts, where they are heavieſt, the Seas about the Poles would ſubſide, and riſing at the Æquator, would overflow the Lands thereabouts.

FROM an experiment made at the beginning of this Lecture it was concluded that the Gravity of every Body is proportional to its Quantity of Matter; and as this is a propoſition of great importance in Natural Philoſophy, it may be proper to explain it a little more fully, and to obviate ſome objections that, to common obſervers, may ſeem to ariſe againſt it. Bodies of different kinds ſeem, at firſt ſight, to be endued with different degrees of weight; thus Lead is ſaid to be heavier than Cork, and Gold to be the heavieſt of all Bodies. Now the reaſon why ſome Bodies are heavier than others of an equal ſize is, that ſome Bodies contain in the ſame bulk, a greater Quantity of Matter than others do. Thus a ball of Lead contains much more Matter than one of Cork of the ſame ſize, for Lead is a much more compact Body than Cork, which abounds with large empty Pores and Interſtices; and therefore ſince the particles of Lead lie cloſer together, it muſt contain more Matter in the ſame bulk than Cork does, and conſequently it ought to be proportionably heavier.

Bodies differ very much in regard to the proportion which their Pores bear to their ſolid Parts, and accordingly are ſaid to be of different Denſities; that being called the denſeſt which contains the greateſt Quantity of Matter in proportion to its bulk. Now as every ſpecies of Matter has a degree of Denſity peculiar to itſelf, if we ſuppoſe many Bodies of different kinds to have the ſame magnitude; each Body will contain a Quantity of Matter proportional to its Denſity, and conſequently each Body will have a different weight, and that in proportion to its Denſity, which weight is therefore called its *ſpecific Gravity*. From hence we ſee how Bodies of the ſame ſize may have different weights, and yet the Gravity of each be ſtill in proportion to its Quantity of Matter.

That Air is heavy like other Bodies, is now well known, and the weight of the Atmoſphere is accurately meaſured by the weight of the Quickſilver that is ſuſtained in the Tube of the Barometer. It is alſo well known, that Smoke aſcends in the open Air for the ſame reaſon that every Body will aſcend in a Fluid that is heavier than itſelf; ſo that we have really no inſtance of any Body abſolutely light or void of weight, but find on the contrary, that all Bodies whatſoever, on or near the ſurface of the Earth, gravitate towards it, and that in proportion to the Quantity of Matter in each of them.

THIS property of Gravitation is not confined to the Matter of which our Earth is formed, but is found to be a general property of all Bodies that comes any way under our obſervation. Sir *Iſaac Newton* has demonſtrated, that the Moon gravitates towards the Earth, and is retained in her Orbit merely by that Force. And ſince the revolution of the Moons round Jupiter and Saturn, and of the primary Planets round the Sun, are Phœnomena, or effects of the ſame kind with the revolution of our Moon round the Earth. He concludes, by the ſecond Rule of Philoſophiſing, that all theſe Effects muſt proceed from like Cauſes, and therefore that the Moons of Jupiter and Saturn gravitate towards their Primaries, and that all the Primary Planets, gravitate towards the Sun. He has alſo ſhewed, that if one Body attracts another with any Force, with the ſame Force does that other Body attract it; ſo that the Earth muſt gravitate towards the Moon, the Sun towards the Planets, and they all towards each other. He has proved likewiſe that the attractive forces of theſe great Bodies act according to one univerſal and invariable Law, which is, that every two of them attract each other with Forces that are directly as their Quantities of Matter, and inverſely as the Square of the diſtance between their Centers. And ſince the Attractions of theſe Bodies are made up of the Attractions of all their Particles, it follows, that every two Particles of Matter muſt attract each other with Forces that are as their Quantities of Matter directly and inverſely as the Square of their Diſtance. The reaſon why we do not perceive the Attraction of any two Bodies towards each other, when they fall freely together near the ſurface of the Earth, is that this Attraction bears the ſame proportion to the weight of the Bodies that the Bodies themſelves do to the whole earth, and therefore it muſt be altogether imperceptible.

Sir *Iſaac Newton's* great diſcovery therefore conſiſts in his having proved that the well-known power which we call Gravity, acting throughout the Solar Syſtem, according to the law above mentioned, is the immediate cauſe which preſerves the Planets and Comets in their Motions round the Sun; and that this Force alone is fully ſufficient to account for all the irregularities of the Lunar Motions, for the Retrogreſſion of the Æquinoctial Points, and for the Tides in our ſeas, whoſe waters gravitate towards the Moon.

All this he has done by ſhewing, from mathematical calculations, that this Force acting on thoſe Bodies muſt neceſſarily produce ſuch effects and appearances, as exactly correſpond with thoſe which, the beſt obſervations aſſure us, do really take place in Nature; ſo that there can remain no doubt of his having aſſigned the true cauſe of all thoſe great Phœnomena.

nomena. Thus has he entirely overthrown all the ſpecious Hypotheſes by which his Predeceſſors endeavoured to explain thoſe matters, and has at length laid open to us the true Syſtem of Nature; in which the wiſe diſpoſition of the great Bodies in the Univerſe, the ſurpriſing regularity of their Motions, and the great ſimplicity and uniformity of the means by which they are effected, appear every way worthy of the infinitely wiſe and almighty Author of all things.

It ſtill remains to be determined, whether theſe Forces of Gravity and Coheſion ariſe immediately from the Will of the Supreme Being, or are the effects of ſome ſecondary or mechanical cauſe, which He is pleaſed to employ in carrying on the operations of Nature? and which, as an immediate inſtrument in his hand, muſt derive its whole efficacy from his Will only. Sir *Iſaac*, from ſome Phænomena, was induced to ſuſpect that the immediate cauſe of Gravity was mechanical; and that there exiſted throughout the Univerſe a moſt ſubtile ætherial fluid, whoſe particles are ſo ſmall as to paſs freely thro' the pores of all Bodies; and that theſe particles are endued with an exceeding ſtrong repelling Force, which makes this fluid vaſtly more rare and more elaſtic than our air, and of conſequence vaſtly leſs able to reſiſt the Motion of Bodies, and much more able to preſs upon groſs Bodies by endeavouring to expand itſelf. This Æther he ſuppoſes muſt, from its repelling force, be much rarer within the denſe Bodies of the Sun, Planets, and Comets than in the empty ſpaces between them; and that in paſſing from them to greater diſtances it grows denſer and denſer perpetually, and thereby cauſes the Gravity of theſe great Bodies towards each other, and of their parts towards the Bodies, every Body endeavouring to go from the denſer parts of this Æther towards the rarer (*d*). All this however he propoſes only as a conjecture, and leaves the truth of it to be determined by future Experiments, and I have mentioned it here only becauſe it is the conjecture of ſo great a Philoſopher. For tho' the late Experiments in Electricity prove the exiſtence of ſome ſuch ſubtile fluid, which eaſily pervades the

(*d*) *Newton*'s Optick's, Query 21.

the denſeſt Bodies, yet I have not heard of any Experiments that prove it to be the cauſe either of Gravity or Coheſion.

Whatever may be the cauſe of Gravity it cannot be improper, in our philoſophic reaſonings about its effects, to conſider it as a quality or property belonging to all Bodies that fall in anywiſe under our obſervation, as our experience ſufficiently warrants us in ſo doing; and ſince this quality cannot in any Body be increaſed or diminiſhed, it muſt be looked upon (by the third Rule of Philoſophiſing) as a quality of all Bodies whatever. This being the caſe, we have from hence an unanſwerable argument for the exiſtence of a *Vacuum*. For if the region of our Air, for inſtance, was filled with any matter that had no empty pores, let this matter be ever ſo ſubtile or ever ſo fluid, it muſt ſtill have a greater Denſity, and conſequently a greater ſpecific Gravity than Quickſilver or Gold, ſo that it would be as impoſſible for even the heavieſt Bodies to deſcend in this Fluid, as it is for Cork to deſcend in Water, or Wood in Quickſilver. And ſince

Bodies, which in the open Air are reſiſted in very different degrees, fall together in an exhauſted Receiver without any ſenſible reſiſtance; we muſt conclude that ſuch a Receiver doth not contain in it any Matter that hath either a ſenſible Gravity, power of reſiſting, or *vis inertiæ*, or in other words, that it doth not contain any ſenſible Matter at all. It is therefore incumbent on thoſe, who aſſert that all Space is equally full of Matter, to tell us what kind of Matter it is that adequately fills a Receiver exhauſted of air, and to ſhew us its properties, ſince it is evidently diveſted of thoſe properties that are common to all the Bodies we know of.

LECTURE IV.

THE
Laws of Motion explained
AND
Confirmed by Experiments.

HAVING given an account of the Properties that are eſſential to Matter, and of the Forces called Attraction and Repulſion; I come now to ſhew how the Motions of Bodies, ariſing from theſe Forces, are regulated according to the Laws laid down in the firſt Lecture, and to confirm theſe Laws by Experiments.

As by the firſt Law of Motion every Body muſt, in proportion to its Quantity of Matter, endeavour to continue in the ſtate it is put into, whether it be of Reſt or uniform rectineal Motion; and as a Body put in Motion by any Impulſe muſt begin to move with ſome certain Velocity, and its Motion muſt be directed towards ſome certain point; if follows, that the Body muſt go on in the ſame right line deſcribing equal ſpaces in equal times, until ſome Force be impreſſed upon it to produce a change. Therefore, when a Body moves in a Curve line, that Curvature muſt neceſſarily proceed from ſome Force which continually acts upon it, and turns from its natural rectilineal Courſe; and whenever that Force ceaſes to act, the Body will move forward in a right line touching the Curve in that point where the Body was when the Force ceaſed to act. The truth of this is ſufficiently teſtified by frequent experience; for if a Stone be moved round in a Sling, by its tendency to move in a right line it will ſtretch the ſtring with ſome Force, and when it is ſet at liberty it will not continue its circular Motion, but will go on in a right line touching the circle made by the revolution of the Sling, until its Gravity draws it from this line towards the Earth.

By the ſecond Law of Motion it appears that any Motion or change of Motion produced in a Body muſt be proportional to, and in the direction of the Force impreſſed. Therefore if a moving Body receives an impulſe in the direction of its Motion, its velocity will be thereby increaſed, if in the contrary direction its velocity will be diminiſhed, tho' its direction will not be changed; but if the Force be impreſſed in a direction oblique to that in which the Body moves, it will be thereby made to change its direction, and move in a line ſituated between the direction of its former Motion and that of the Force impreſſed. And if a Body at reſt receives two Impulſes at the ſame time from Forces whoſe directions do not coincide, it will, by their joint action, be made to move in a line that lies between the directions of the Forces impreſſed. For inſtance, if on the Body A (fig. 4) a Force be impreſſed ſufficient to make it move, with an uniform velocity, to the point B in a ſecond of time, and if another Force be alſo impreſſed on the Body which alone would make it move to the point C in the ſame time; the Body by means of theſe two Forces will deſcribe a line whoſe poſition may be thus determined. Complete the parallelogram A B D C, whoſe ſides A B and A C are by the ſuppoſition proportional to the Forces impreſſed in their directions, draw the diagonal A D; the Body will move along this line and will deſcribe it in the ſame time in which it would have deſcribed either of the ſides A B or A C ſeparately. For ſince the Force impreſſed on the Body to carry it to the point C, is not directed either towards the line B D or from it, but acts in a direction parallel to it, that Force cannot either accelerate or retard the velocity with which the Body approaches the line B D, the Body muſt therefore arrive at the line B D in the ſame time it would have done if no other Force had been impreſſed on it but that in the direction A B, and therefore at the end of one ſecond it will be found ſomewhere in the line B D. In the ſame manner, ſince the Force impreſſed in the direction A B, which is parallel to C D, is neither directed towards C D nor from it, it cannot prevent the Body from arriving at the line C D in the ſame time it would have done, had no other Force been impreſſed than that in the direction A C; and therefore the Body will, at the end of one ſecond, be found ſomewhere in the line C D; conſequently, at the end of the ſame time the Body will be found in both lines, that is, at the point D, the concourſe of thoſe lines. And ſince, from the joint impreſſions of the Forces, it muſt have ſet out from the point A, with a rectilineal Motion, it muſt, during its Motion have deſcribed the whole line A D the diagonal of the parallelogram C A B D.

To confirm this by an Experiment let three ivory balls, of an equal ſize, be ſuſpended from three pins, by ſtrings of equal lengths, and let the middle ball reſt over one angle of a wooden ſquare: then let each of the extreme balls be let fall ſeparately, from the ſame height, ſo as to ſtrike the middle ball in the direction of one ſide of the ſquare; the middle ball will, by each of the ſtrokes made ſeparately, be moved over that ſide of the Square that lies in the direction of the ſtroke; but if the two balls be at the ſame time let fall from equal heights, ſo that they may ſtrike the middle ball at once, and in the directions of the two ſides of the Square, the middle ball will, by the joint Forces impreſſed at the ſame time, be driven over the diagonal of the Square.

From what has been ſaid it appears that when a Body is acted upon at the ſame time by two Forces that are to each other as the ſides of a parallelogram A B and A C, and act in the directions of thoſe ſides, it will deſcribe the diagonal in the ſame time in which it would be made to deſcribe the reſpective ſides by each of thoſe Forces ſeparately, Therefore the velocity wherewith the Body moves along the diagonal, is to the velocity wherewith it would move along the ſides, when acted upon by each Force ſingly, as the diagonal to each ſide reſpectively. And ſince the Force which gives an impulſe to a Body is proportional to the velocity with which it makes the Body move, we may ſay that the Force by which the Body is impelled along the diagonal, is to either of the impelling Forces, as the diagonal is to the ſide in the direction of which that Force acts; and therefore it will be to the ſum of the impelling Forces, as the diagonal to the ſum of the ſides.

As the line A D may be made the diagonal of numberleſs parallelograms, a Body may be made to move along the line A D by numberleſs pairs of Forces acting in the directions of the ſides of thoſe parallelograms, and proportional thereunto. So that the Force of a Body moving along the line A D may be conſidered as compounded of, or produced by, two Forces acting in any oblique directions, ſuch as A B and A C, or A E and A F, ſince the very ſame Motion would ariſe from ſuch a compoſition. And conſequently the Force of a Body moving in the line A D may be reſolved into two Forces acting in thoſe oblique directions. From this compoſition and reſolution of Forces or Motions, we may deduce the following Theorem, which is of great uſe in Mechanicks, for determining the effects of Forces acting againſt each other in oblique directions.

If three Forces or Powers acting on a Body in oblique directions be to each other reſpectively as the ſides of a triangle drawn parallel to their directions, thoſe Powers will balance one another and keep the Body at reſt.

Let three Powers, drawing the Body A (fig. 4.) in the directions A B, A C and A G, be to one another as the ſides of the triangle A B D, which are reſpectively parallel to their directions, they will balance each other, and the Body will remain unmoved. For taking A C equal to B D, let the parallelogram A B D C be completed, then, from what was ſaid of the compoſition of Forces, the Powers which draw the Body A in the directions of the ſides A B and A C, will, by their joint action, urge that Body from A towards D, in the direction of A D, with a Force which is to that of either of theſe Powers, as A D to either of the ſides reſpectively; but the Body is drawn in the oppoſite direction from A towards G, by the third Power, whoſe Force, by the ſuppoſition, is to that of either of the other Powers as the line A D is to A B or A C reſpectively; therefore the Force of this third Power is equal to the joint action of the other two Powers; conſequently the three Powers muſt balance each other, and the Body A, being drawn equally in oppoſite directions, will remain unmoved. In the ſame manner we may prove that the joint Force of any two of theſe Powers acts in a direction oppoſite to that of the third, and is equal thereunto; and therefore any three Powers which are to one another as the ſides of a triangle, and act in directions parallel to theſe ſides, will deſtroy each others effect, and remain in Æquilibrio. And from hence it follows, that when three Powers, acting againſt each other in oblique directions, are in Æquilibrio, they will be to each other as the ſides of a triangle which are reſpectively parallel to their lines of direction.

To confirm this by an Experiment let the ſides of a triangle A B D, drawn on an horizontal Plane, be as two, three and four; and let A C be parallel to the ſides B D, and let D A be continued towards G; let then three ſmall cords be tied together at A, and ſtretched over three pulleys at the points G, H and L, ſo that one cord may cover the line A G, another the line A B, and the third the line A C; this being done, if a weight of four ounces be hung to the cord which covers A G, and one of three ounces to that which covers A B, and one of two ounces to that which covers A C, theſe three weights, which are to one another as the ſides of the triangle that are parallel to their lines of direction, will continue in Æquilibrio.

THE third Law of Motion, which ariſes from the Inertneſs of Matter, is, that reaction is always equal to action, and in a direction contrary thereto; or that whatever Motion one Body can by its action produce in another, the ſame Motion will be produced in it by the reaction of that other, and in a contrary direction.

IN my Lecture on Gravity, I obſerved, that it has been demonſtrated that each of the Planets gravitates towards the Sun, and the Sun towards each of them with a reciprocal and equal Force. The ſame equality of Attraction obtains likewiſe between the Loadſtone and Iron; for if a Loadſtone and a piece of Iron be ſet on two pieces of Cork, and made to float on water near each other, they will come together with equal quantities of Motion, that is, if their weights are equal, they will approach with equal velocities, or if the weight of the Loadſtone be double that of the Iron, the velocity of the Iron will be double that of the Loadſtone, ſo that the attractions between thoſe Bodies are equal and reciprocal. When two light Bodies are ſuſpended near each other, if one be electrified and the other not, they will equally and mutually attract each other and come together.

If a man, placed in a boat, draws another boat by means of a rope, the one wherein the man is placed will be equally drawn with the other, and the two boats will come together with equal quantities of Motion. So that if they are of the ſame weight, ſize and ſhape, they will move with equal velocities; but if one boat be in weight double the other, it will move with only half the velocity of the other, and they will ſtill approach each other with equal quantities of Motion. This will appear by the following Experiment.

Let a cord be made faſt to one end of a ſmall boat, and let it paſs over a pulley fixed to the end of another ſmall boat of the ſame ſize and ſhape, and let a weight be tied to the end of the cord, and hang in the water, then let the boats be placed at ſuch a diſtance, as that the cord may be ſtretched ; when the boats are let go, the weight will deſcend and thereby draw the boat to whoſe end the cord is faſtened towards the other, and and at the ſame time the other will move towards it. And the quantities of Motion in both boats will be equal, for their velocities will be equal when their weights are ſo, and when their weights are unequal, their velocities will be inverſely as their weights, for when they meet, the Spaces they have deſcribed will be found to be in that proportion.

As Action and Reaction are equal in all attractions, ſo are they likewiſe equal in reſpect to ſtrokes or impulſes made by Bodies one upon another ; for the Force of two Bodies, ſtriking each other, affects equally the Motion of both, and produces equal changes therein towards contrary Parts ; ſo that when one Body ſtrikes againſt another and communicates Motion to it in a certain direction, juſt ſo much Motion does the ſtriking Body loſe in that direction, or juſt ſo much Motion is there communicated to it in the contrary direction. I ſhall therefore confirm this third Law of Motion, by ſhewing that what I have now ſaid, holds true in all caſes, from Experiments made by the colliſion of Bodies both non-elaſtic and elaſtic: but firſt I muſt lay before you the method of making theſe Experiments.

THE Bodies that are to ſtrike each other muſt be formed into Spheres; and then ſuſpended from two pins by threads of an equal length, and in ſuch a manner, as that, when they hang freely they may juſt touch one another, and that their centres and point of Contact may lie in a right line parallel to the Horizon. Now that you may underſtand the manner of eſtimating the velocities with which theſe Bodies are made to ſtrike each other, I muſt lay down ſome things concerning the Motion of Bodies thro' the arches of Circles the truth of which ſhall be demonſtrated when I come to treat of Pendulums.

AND firſt, all the arches of a Circle, provided they be not large, are deſcribed in equal times by pendulous Bodies deſcending along them; and therefore if two pendulous Bodies be let fall at the ſame time, one from C and the other from E in the periphery of a Circle whoſe radius is A B (fig. 5.) they will both arrive at the loweſt point B at the ſame time; and the ſtroke of the ſubſequent Body upon the preceding will be made at B: and for the ſame reaſon if one be let fall from C or E, and the other from D or F, they will meet and ſtrike one another at B.

SECONDLY; the velocity which a Body acquires in falling thro' the arch of a Circle, is as the cord of the arch; that is, the velocity of a Body which has fallen from C to B, is to the velocity of a Body that has fallen from E to B, as the cord C B to the cord E B. And here I muſt obſerve to you that when, in the following Experiments, I ſpeak of a Body falling from, or riſing to any height, as four, ſix or ten inches, I mean it of a Body's falling thro', or moving up, an arch whoſe cord is of ſuch a length.

THIRDLY; the velocity wherewith a Body begins to riſe up thro' the arch of a circle, is as the cord of the arch which the Body deſcribes in its aſcent. Thus the velocity wherewith a Body begins to move from the point B towards D, if it aſcends as high as D, is as the chord B D, but if it riſes only to F, the velocity is as the cord B F: ſo that in the Experiments the cords of the arches through which the Bodies deſcend expreſs the velocities of the Bodies in the point B, at the time of the ſtroke; and the cords of the arches, thro' which the Bodies aſcend after the ſtroke, expreſs the velocities of the Bodies immediately after the ſtroke.

THESE things being premiſed, I now proceed to the Experiments made firſt with Bodies that are non-elaſtic, that is, ſoft Bodies whoſe parts yield to a ſmall Force, and, when compelled to change their ſhape, do not again return to it.

LET two equal balls of ſoft clay or putty be ſuſpended in the manner abovementioned, and while one of them continues at reſt, let the other fall upon it from the height of ſix inches; ſince the Bodies are void of elaſticity they will not ſeparate after the ſtroke, but both together aſcend with a common velocity to the height of three inches. Therefore the common velocity of both Bodies after the ſtroke is but half the velocity which the ſtriking Body had before the ſtroke, and conſequently, ſince the Bodies are equal, each of them will have half the quantity of Motion which the ſtriking Body had before the ſtroke; ſo that the ſtriking Body has loſt, by the ſtroke, half its quantity of Motion, and juſt ſo much has the quieſcent Body gained.

Let now one of the equal balls fall from the heigth of nine inches, and the other from the height of three in a contrary direction, the Motions of thoſe Bodies before the ſtroke are reſpectively nine and three, and their difference ſix; after the ſtroke they will aſcend together in the direction of the ſtriking, that is the ſtrongeſt, Body to the height of three inches; therefore the Motion of each Body will then be three, and the ſum of their Motions ſix, which is the ſame with the difference of their Motions before the ſtroke; conſequently the ſtriking Body, whoſe Motion before was nine, has, by the ſtroke, loſt ſix; and the other Body which is turned back with three parts of Motion, muſt have had a Motion as ſix communicated to it by the ſtroke, by one half of which its former Motion (which was three) is deſtroyed, and with the other half it moves in a direction contrary to its former one, ſo that the Motion loſt by the ſtriking Body is the ſame with that communicated to the other.

It is evident in all caſes, when the Bodies come together moving in oppoſite directions, that the difference of their Motions before the ſtroke will be the ſum of their Motions after it. For the ſtrongeſt Body muſt deſtroy the whole Motion of the weaker, and by the reaction of the weaker, is muſt loſe as much of its own Motion; therefore at the inſtant of the ſtroke there will remain only the difference of their Motions in the ſtriking Body and with this it will move on its former direction, carrying the weaker Body along with it.

If therefore the Bodies moving in oppoſite directions come together with equal quantities of Motion, they will upon the ſtroke loſe all their Motion, and continue at reſt. To prove which by an Experiment,

LET two equal balls of clay fall at the ſame time from equal heights, and in oppoſite directions, upon the ſtroke they will ceaſe to move. And the ſame thing will happen when the balls are unequal, provided the heights from which they fall are reciprocally proportional to their quantities of Matter, for then their quantities of Motion are equal; for inſtance, if the balls be as one and two; let the former fall from the height of ſix inches and the latter from the height of three in an oppoſite direction, and upon their meeting they will loſe all Motion; each Body communicating to the other a quantity of Motion equal to what that other had before the ſtroke and in an oppoſite direction; and therefore theſe Motions muſt neceſſarily deſtroy each other. From theſe Experiments it appears that whatever Motion one non-elaſtic Body communicates to another, ſo much does it loſe itſelf by the reaction of that other.

I AM now to ſhew that when two elaſtic Bodies ſtrike together, the Motion loſt by one will be equal to that communicated to the other. Bodies are ſaid to be elaſtic whoſe parts yield and give way when preſſed, and reſtore themſelves again to their former ſituation when the preſſure is removed; and if the Force wherewith they reſtore themſelves be exactly equal to the Force whereby they are bent inwards, then the Bodies are ſaid to be perfectly elaſtic; and ſuch are all thoſe Bodies ſuppoſed to be with which Experiments are made for confirming the theory relating to the colliſion of elaſtic Bodies. But as there is not perhaps in nature any Body perfectly elaſtic, where the Experiments vary a little from the Theory, ſuch variation muſt be aſcribed rather to the want of perfect Elaſticity in the Bodies, than to any error in the Theory itſelf.

IF of two Bodies, perfectly elaſtic, one be at reſt and the other in Motion, or if they both move the ſame or contrary ways, ſo that they may ſtrike each other directly; the quantity of Motion which is acquired by one Body will be equal to that which is loſt by the other.

For it appears, from what has been already ſaid, that the quantity of Motion communicated to one Body merely by the ſtroke, will be equal to that which the ſtriking Body loſes by the ſtroke when the Bodies are non-elaſtic; now I ſhall ſhew that theſe quantities of Motion will be juſt doubled by means of the Elaſticity, and conſequently they muſt ſtill be equal. Since the Bodies are ſuppoſcd to be perfectly elaſtic, their parts when bent inwards by the ſtroke, will reſtore themſelves to their former ſituation with a Force equal to that which bends them in; but this Force is meaſured by the quantity of Motion which the ſtriking Body communicates to the other; therefore after the ſtroke this elaſtic Force will throw the Bodies different ways, and each with a quantity of Motion equal to that which the ſtriking Body communicates to the other, which added to the communicated Motion will make that double, being in the ſame direction therewith, and which ſubducted from the Motion of the ſtriking Body (as contrary thereto) will make the loſs double which it ſuſtained by the ſtroke. And therefore the whole Motion which one Body acquires will be equal to that which the other loſes; or in other words, the Force of the two Bodies ſtriking each other will equally affect the Motion of both and produce equal changes therein towards contrary parts; ſo that the action of the ſtriking Body, and the reaction of the other upon it are equal in all caſes. What has been now proved will be confirmed by the following Experiments.

If two equal balls of ivory be ſuſpended as were thoſe of clay, and be let fall from equal heights in oppoſite directions, they will be both reflected back by the ſtroke, and will riſe to almoſt the ſame height from which they fell. When the balls were void of elaſticity after the ſtroke they continued at reſt, each having communicated its whole Motion to the other, which communicated Motion in each Body ſerved only to deſtroy its former Motion being equal and in a contrary direction thereto; now it is plain that the balls will merely by the ſtroke communicate the ſame Motion to each other, whether they be elaſtic or not; and therefore ſince the balls deſtroy each other's Motion by the ſtroke, and recover very nearly their whole Motions by means of their elaſticity, (each Body being thrown back with almoſt the ſame velocity it had before the ſtroke) we may conclude that the parts, when compreſſed by the ſtroke, reſtore themſelves with a Force nearly equal to the compreſſing Force, and which may therefore be meaſured by the Motion communicated.

LET a ball of ivory fall from any height and ſtrike an equal ball at reſt; the ball which receives the ſtroke will aſcend to nearly the ſame height from which the other fell, and the other will remain at reſt; ſo that the ſtriking Body loſes its whole Motion, and the quieſcent Body acquires as much. In this caſe when the

Bodies were non-elaſtic, the ſtriking Body communicated only half its Motion to the quieſcent one, and loſt ſo much itſelf.

We ſee therefore, from theſe Experiments, that the Elaſticity of the Bodies, were it perfect, would double the quantities of Motion which are communicated and loſt merely by the ſtroke, and that it equally increaſes them both, tho' it is imperfect; and conſequently we may conclude, that whenever two Bodies, either non-elaſtic or elaſtic, ſtrike each other, the Force of the ſtroke will make equal changes in the Motions of each, or that equal quantities of Motion will be communicated to each in oppoſite directions; ſo that the ſum of their Motions, when they move the ſame way, or the Difference of their Motions, when they move contrary ways, taken in the Direction of the ſtriking Body, will be the ſame after the ſtroke as it was before.

If, inſtead of one, there be two, three, or more quieſcent balls contiguous to each other, that which is fartheſt removed from the ſtriking ball will fly off with the velocity of the ſtriking ball, leaving it and all the intermediate balls at reſt. For as the ſtriking ball imparts all its Motion to the firſt of the quieſcent balls, ſo does that in like manner to the ball that lies next beyond it, and that again to the third, and ſo on, till the laſt ball meeting no other to reſiſt it, flies off with all the Motion which the ſtriking ball had before the ſtroke.

I SHALL add one Experiment more, by which we may ſee that an elaſtic Body, by ſtriking another, may communicate to it more Motion than it had itſelf. If an ivory ball falls from the height of nine inches and ſtrikes another at reſt that has double its quantity of Matter; after the ſtroke the ſmaller ball will be reflected back to the height of three inches, and the larger ball will riſe to the height of ſix inches, ſo that its velocity ſix multiplied into two, its quantity of Matter, gives twelves for its quantity of Motion after the ſtroke, which is three parts of Motion more than the ſtriking body had before the ſtroke. To account for this, we muſt conſider, that if the balls were non-elaſtic, the whole Motion of the ſtriking ball, which was nine, would after the ſtroke be divided between the balls, in proportion to their quantities of Matter, becauſe they would then move on together with a common velocity; therefore the larger ball would merely by the ſtroke acquire two thirds of the leſſer ball's Motion, that is, ſix, and ſo much would the leſſer ball loſe; ſince therefore, by means of the Elaſticity, the Motion communicated and loſt is doubled, the larger ball muſt acquire twelve parts of Motion, and therefore muſt riſe to the height of ſix inches; and the leſſer ball muſt loſe twelve parts of Motion, which is three more than it had before the ſtroke, therefore it will be reflected back to the height of three inches; for a Body is properly enough ſaid to loſe more Motion than it had in a certain direction, when its Motion is deſtroyed, and it is made to move in the contrary direction. Hence it follows, whenever a ſmaller elaſtic ball ſtrikes a larger one at reſt, that there will be an augmentation of Motion, and that the exceſs of Motion in the larger ball, above the Motion which the ſmaller has before the ſtroke, will be ever equal to the Motion wherewith the ſmaller ball is reflected back after the ſtroke, as is evident from what has been ſaid. If therefore Motion be communicated from a ſmaller elaſtic Body to a larger, by means of ſeveral intermediate Bodies, each larger than the other, the Motion will be augmented in each of them, and the Motion of the laſt will greatly exceed that of the firſt.

FROM the increaſe of Motion in elaſtic Bodies, a reaſon may be drawn for the augmentation of ſound in ſpeaking-trumpets; for as the ſpeaking-trumpet is narroweſt at the mouth-piece, and thence widens and enlarges continually to the extremity, the air within it, which is an elaſtic Fluid, may be conſidered as divided into a great number of cylindrical Bodies, of very ſmall but equal altitudes, the baſis of the firſt being equal to the aperture of the trumpet, to which the mouth is applied, and the baſes of the reſt increaſing one above another as they are more and more removed from the mouth; upon which account the Motion, that is impreſſed by the Force of the Voice on the firſt cylindrical Body of Air, grows greater in the ſecond, and greater ſtill in the third, and ſo on, 'till at length, at the exit of the Tube, it becomes ſo great as to magnify the ſound very conſiderably.

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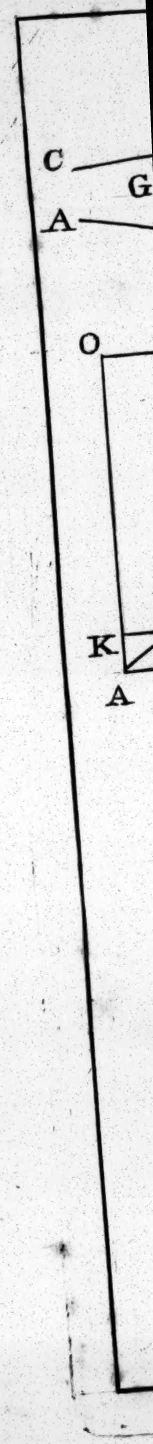

Fig. 1.

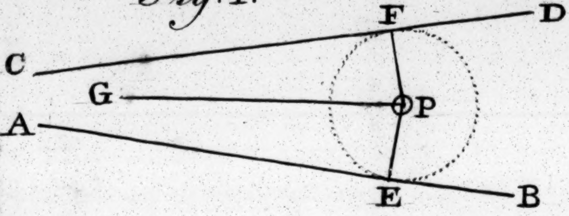

Fig. 2.

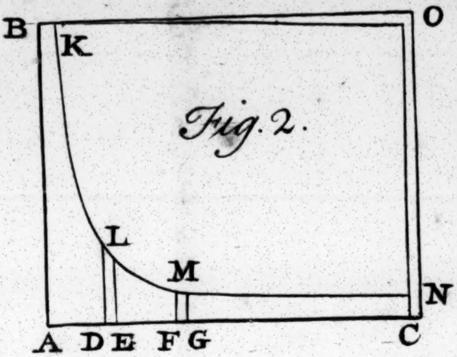

Fig. 3.

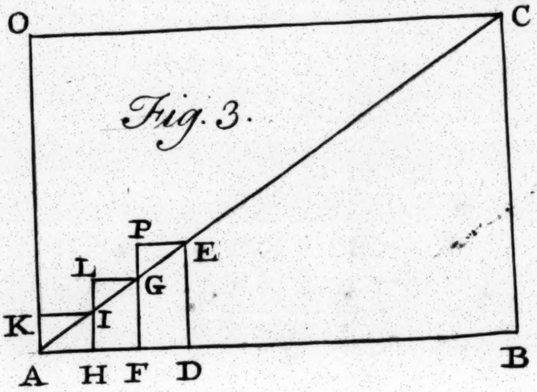

Fig. 4.

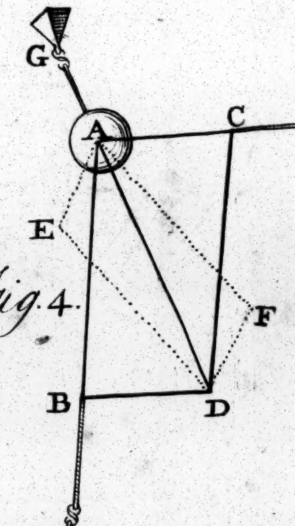

Fig. 5.